Examination Statistical Methods in Physics
Monday, April 9 2018, 18:30-21:30

Before you start, read the following:

- Write your name and student number on top of each page of your exam;
- Illegible writing will be graded as incorrect;
- Annexes:
  - Integral of the Standard Normal distribution
  - Quantiles of the Chi-squared distribution

Problem 1 (15 Points)

Using method of moments estimate parameter θ of the normal distribution with mean equal to θ and variance equal to:

a) 2θ **(5 points)**;

b) θ^2 **(10 points)**.

In both cases use the first and second algebraic moments. (You can use known expressions for the first and second algebraic moments of the normal distribution).

Problem 2 (20 Points)

Using method of maximum likelihood find parameter θ of the normal distribution with mean equal to θ and variance equal to θ^2.

Problem 3 (25 Points)

Let X_1 – a sample of size one. The main hypothesis is that X is uniformly distributed in the interval $[0, 1]$. An alternative hypothesis is that X comes from the exponential distribution with mean equal to one. Construct the most powerful hypothesis test with level of significance α **(15 points)**. What is the power of the test **(10 points)**. (Hint: use Neyman-Pearson test)

Problem 4 (25 Points)

Let X_1, ..., X_n, $n = 10$ – a sample of a normally distributed data with mean equal to a and variance of σ^2. Value of a is known. Derive expression for the 90% confidence interval for estimation of variance using statistics $S_1^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - a)^2$ as a point estimator.

Problem 5 (15 Points)

A process for making steel pipe is under control if the diameter of the pipe has mean 3.0 cm with standard deviation of no more than 0.0250 cm. To check whether the process is under control, a random sample of size $n = 30$ is taken each day and the null hypothesis $\mu = 3.0$ is rejected if $\overline{X}_n$ is less than 2.9960 or greater than 3.0040. Find

a) the probability of type I error; **(7.5 points)**

b) the probability of type II error when $\mu = 3.0050$ cm. **(7.5 points)**

Assume $\sigma = 0.0250$ cm.

Table 1: Integral of the Standard Normal distribution: $\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} dx$.

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.00	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.10	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.20	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.30	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.40	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.50	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.60	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.70	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.80	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.90	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.00	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.10	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.20	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.30	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.40	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.50	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.60	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.70	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.80	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.90	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.00	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.10	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.20	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.30	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.40	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.50	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.60	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.70	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.80	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.90	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.00	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.10	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.20	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.30	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.40	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.50	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.60	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.70	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.80	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.90	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table 2: Quantiles of the Chi-squared distribution: $\int_0^x \chi^2(NDF)dx = \alpha$

NDF; α	0.005	0.010	0.025	0.050	0.900	0.950	0.975	0.990	0.995
1	0.000	0.000	0.001	0.004	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	9.236	11.070	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	36.741	40.113	43.195	46.963	49.645
28	12.461	13.565	15.308	16.928	37.916	41.337	44.461	48.278	50.993
29	13.121	14.256	16.047	17.708	39.087	42.557	45.722	49.588	52.336
30	13.787	14.953	16.791	18.493	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	63.167	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	74.397	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	85.527	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	107.565	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	118.498	124.342	129.561	135.807	140.169